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On the Accuracy of the Complex Image Method

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1 Introduction

Sommerfeld Integrals (SIs) are a type of oscillatory integral that is used to describe the spatial Greens' functions of dipole sources radiating in layered media. They may be used in conjunction with moment methods to model scattering and radiation in such media.

The Complex Image (CI) method is a rapid but approximate technique used to calculate spatial Greens' functions without recourse to time-consuming numerical integration of SIs; this greatly decreases the fill time for the moment method interaction matrix.

The CI method operates by approximating part of the spectral Greens' function as a sum of complex exponentials; thereby allowing one to inverse transform it in to a closed-form spatial Greens' function. This process is generally an order of magnitude faster than numerical integration.

The main problem with the CI method is that there is no way of checking the accuracy of the spatial Greens' functions it is used to calculate, until they have been derived and compared with established techniques—like numerical integration or asymptotic evaluation. This paper introduces for the first time a new variant of the CI method that uses a new path in the complex plane. It is used to calculate the vector potential Greens' function of an infinitesimal Vertical Magnetic Dipole (VMD), radiating above a half-space. The new and standard CI methods are compared with asymptotic techniques. This comparison is used to explain why the standard CI method fails under certain conditions, and when one should consider employing the new method instead.

2 Modeling a VMD above a Half-space

Consider the case of an infinitesimal VMD in free space radiating above a half-space of permittivity $\epsilon_1 = \epsilon_r \epsilon_0$. The vector potential dyadic Greens' function for this problem geometry is $\bar{\bar{G}}_E = \hat{z}\hat{z}G_{EE}^{zz}$ where:

$$G_{EE}^{zz}(\rho, z; \rho', z') = \frac{1}{4\pi} \int_{-\infty}^{\infty} \tilde{G}_{EE}^{zz}(k_\rho, z, z') H_0^{(2)}(k_\rho |\rho - \rho'|) k_\rho dk_\rho. \quad (1)$$

(ρ, z) and (ρ', z') are the cylindrical observation and source coordinates respectively; \hat{z} is perpendicular to the plane of the interface and k_ρ is the wave-number in the ρ direction.

The spectral Greens' function \tilde{G}_{EE}^{zz} is:

$$\tilde{G}_{EE}^{zz}(k_\rho, z, z') = \frac{1}{j2k_{z0}} \left[e^{-jk_{z0}|z-z'|} + \tilde{R}_{10}^{TE} e^{-jk_{z0}(z+z')} \right], \quad (2)$$

where the wave-number in free space $k_0^2 = k_{z0}^2 + k_\rho^2$.

The TE plane-wave spectral reflection coefficient \tilde{R}_{10}^{TE} is:

$$\tilde{R}_{10}^{TE} = \frac{k_{z0} - k_{z1}}{k_{z0} + k_{z1}}. \quad (3)$$

2.1 Asymptotic Model

As ρ and $(z + z')$ increase asymptotic techniques can be used to derive expressions for $G_{\vec{r}}^{z,z'}$ such as that given in equation 4:

$$G_{\vec{r}}^{z,z'} = \frac{e^{-j k_0 R_0}}{4\pi R_0} + \hat{R}_{10}^{TE}(k_{\rho,z'}) \frac{e^{-j k_0 R_1}}{4\pi R_1} + G_{\vec{r}}^{z,z'}{}_{iw}, \quad (4)$$

where

$$R_1 = \sqrt{(\rho - \rho')^2 + (z + z')^2} \quad (5)$$

and

$$k_{\rho,z'} = k_0 \frac{\rho}{\sqrt{\rho^2 + (z + z')^2}}. \quad (6)$$

The first term in equation 4 is the direct term of the VMD and is obtained by analytical integration. The second term is the reflected term derived using the *method of stationary phase* [1], a simple asymptotic technique that is valid when the source and observer are not close to the interface. These two terms represent two spherical waves—one emanating from the source and one from its image (which is modified by $\hat{R}_{10}^{TE}(k_{\rho,z'})$).

From equations 3 and 6 it can be seen that as $(z + z') \rightarrow \infty$ and $\rho \rightarrow \infty$, $\hat{R}_{10}^{TE}(k_{\rho,z'})$ tends to -1 . Therefore the first two terms in equation 4 cancel each other out, resulting in the field along this interface region being dominated by $G_{\vec{r}}^{z,z'}{}_{iw}$.

$G_{\vec{r}}^{z,z'}{}_{iw}$ may be derived using complicated and problem dependent asymptotic techniques and represents a *lateral wave* [1]. Lateral waves are inhomogeneous waves that have an algebraic decay of $\frac{1}{\rho^2}$ in the ρ direction, and are evanescent as one moves away from the interface.

2.2 Standard Complex Image Model

The standard CI method proceeds by parametrically sampling the TE plane-wave spectral reflection coefficient, \hat{R}_{10}^{TE} , along Path 1 (see fig. 1) in the k_{z0} plane [2]. The parameter T_0 is chosen in order that Path 1 avoids the branch points at $k_\rho = k_0$ and $k_\rho = k_1$ (marked with an x in fig. 1.)

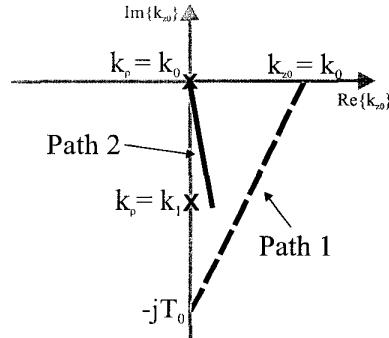


Figure 1: Sample Paths on the Complex k_{z0} Plane.

The samples of \hat{R}_{10}^{TE} are then approximated by a sum of N complex exponentials of a real variable, using a system identification technique such as the SVD-Prony's method [3, 4] (any error in this approximation introduces an error into the final Greens' Function), which is then transformed back into a function of the complex variable k_{z0} [2]:

$$\hat{R}_{10}^{TE} \approx \sum_{i=1}^N a_i e^{-b_i k_{z0}}. \quad (7)$$

An approximate closed-form expression for $G_{\vec{r}}^{zz}$ can now be derived using equations 1, 7 and the Sommerfeld identity [1]:

$$G_{\vec{r}}^{zz} = \frac{e^{-jk_0 R_0}}{4\pi R_0} + \sum_{i=1}^N a_i \frac{e^{-jk_i R_i}}{4\pi R_i} \quad (8)$$

where

$$R_0 = \sqrt{(\rho - \rho')^2 + (z - z')^2} \quad (9)$$

and

$$R_i = \sqrt{(\rho + \rho')^2 + (z + z' + j b_i)^2}. \quad (10)$$

When compared with numerical integration, the standard CI method only produces accurate results for a limited range of source and observer coordinates; for example in fig. 2 it fails as ρ increases above a certain threshold value. In this example the model parameters were $\epsilon_r = 6$, *frequency* = 1GHz and $z + z' \approx 0.25\lambda$. 100 samples and 7 complex image terms were used to approximate \hat{R}_{10}^{TE} .

The inaccuracy of the standard CI method in this example can be explained as follows: the terms in equation 8 represent a spherical wave emanating from the source and a summation of spherical waves emanating from complex locations in the lower half-space, modified by complex residues. In the spatial domain, the CI method therefore characterizes the source's interaction with the interface as a summation of spherical waves—these spherical waves are not, however, an ideal basis to model a lateral wave.

The standard CI method works well enough in regions where the lateral wave is not strongly excited, such as when the source and/or observer are far from the interface. However, it fails when one moves along the interface in the ρ direction because ρ gradually dominates the poles in equation 10:

$$\rho \rightarrow \infty \Rightarrow R_i \rightarrow \rho. \quad (11)$$

The information about \hat{R}_{10}^{TE} that is modeled by the poles in equation 7 is then lost, causing significant errors in the spatial Greens' function.

2.3 New Complex Image Model

The new CI method proposed in this paper samples \hat{R}_{10}^{TE} along Path 2 in the complex k_{z0} plane (see fig. 1). Path 2 starts at the branch point at $k_\rho = k_0$ and terminates just off the imaginary axis near the branch point at $k_\rho = k_1$. This path concentrates the approximation process on the behavior of \hat{R}_{10}^{TE} along the portion of the k_{z0} plane that describes waves that excite lateral wave modes. Therefore the new technique is much more robust along the interface (see fig. 2).

This new CI method shows for the first time how the integration of equation 1 can be performed over a limited range of the variable k_ρ , to improve the accuracy and robustness, in a predetermined region of space, of the CI spatial Greens' function $G_{\vec{r}}^{zz}$.

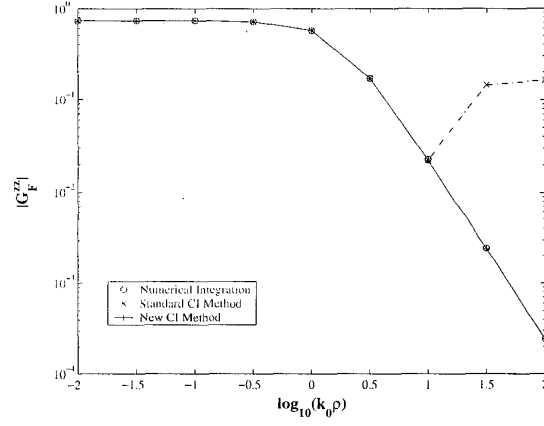


Figure 2: $|G_z^z|$ versus $\log_{10}(k_0 \rho)$ for a VMD above a Half-Space

3 Conclusions

The far-field asymptotic expressions for the vector potential Greens' function of an infinitesimal Vertical Magnetic Dipole (VMD), radiating above a half-space, are used to explain the failure of the standard CI method, when the source and observer are near to the interface—but separated by a large lateral distance. In these types of problems the field at the observation point is dominated by a lateral wave that is not modeled accurately by the standard CI method. It is shown for the first time that by applying the CI approximation process over a different path in the k_{x0} plane, the accuracy and robustness of the CI method for such a problem geometry can be improved.

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